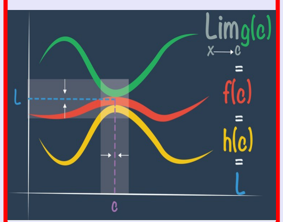
Math 261

Fall 2022

Lecture 32



Math 261
Class Quiz 14

Name:________________________

No Work ⇔ No Points

Use Pencil Only ⇔ Be Neat & Organized

1. (6 points) Find the equation of tangent line to the curve $\sin(x+y) = 2x - 2y$ at (π, π) .

$\sin(\pi+\pi) \stackrel{?}{=} 2\pi - 2\pi$

$\sin 2\pi = 0$ ✓

$\cos(x+y)\cdot\left[1+\frac{dy}{dx}\right] = 2 - 2\frac{dy}{dx}$

Plug in (π,π), Replace $\frac{dy}{dx}$ with m

$\cos 2\pi \,(1+m) = 2-2m$

$1+m = 2-2m \qquad m = \frac{1}{3}$

$y - \pi = \frac{1}{3}(x-\pi)$

1. $\boxed{y = \frac{1}{3}x + \frac{2\pi}{3}}$

2. (8 points) Show that the curves given by $y = ax^3$ and $x^2 + 3y^2 = b$ are orthogonal at each point of intersection.

$y = ax^3 \qquad y' = 3ax^2$

$x^2 + 3y^2 = b$

$2x + 6y\frac{dy}{dx} = 0 \qquad y' = \frac{-2x}{6y}$

$y' = \frac{-x}{3y}$

Show product of slopes is -1.

$\cancel{3}ax^2 \cdot \frac{-x}{\cancel{3}y} = -\frac{ax^3}{y} = \boxed{-1}$

2. ✓

3. (6 points) If $f(x)$ is an even function, show that $f''(x)$ is an even function.

$f(x)$ is even

$f(-x) = f(x)$

$f'(-x)\cdot -1 = f'(x)$

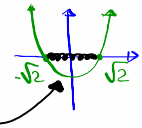
$f''(-x)\cdot -1 \cdot -1 = f''(x)$

$f''(-x) = f''(x)$

$\therefore$ $f''(x)$ is even funct.

3. ________

$f(x) = x\sqrt{2-x^2}$



Domain $\quad 2-x^2 \geq 0 \qquad x^2-2 \leq 0$

$[-\sqrt{2}, \sqrt{2}]$

$f(-x) = -x\sqrt{2-(x)^2} = -x\sqrt{2-x^2} = -f(x)$

$f(x)$ is an odd function $\Rightarrow$ Symmetry $\rightarrow$ Origin

x-Int $\quad f(x)=0 \rightarrow x\sqrt{2-x^2}=0 \rightarrow x=0$, $x=\pm\sqrt{2}$

$(0,0), (\sqrt{2},0), (-\sqrt{2},0)$

Y-Int $(0,0)$

$f(x) = x(2-x^2)^{1/2}$

$f'(x) = 1(2-x^2)^{1/2} + x\cdot\frac{1}{2}(2-x^2)^{-1/2}\cdot -2x$

$= (2-x^2)^{1/2} - x^2(2-x^2)^{-1/2}$

$= (2-x^2)^{-1/2}\left[(2-x^2)^1 - x^2\right]$

$= (2-x^2)^{-1/2}(2-x^2-x^2) = (2-x^2)^{-1/2}\cdot 2(1-x^2)$

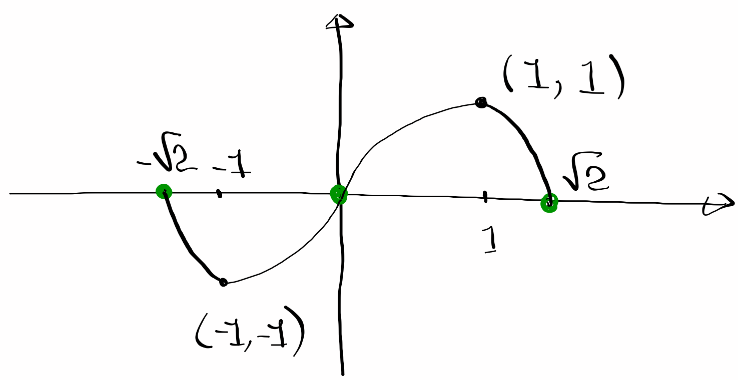
$f'(x) = \dfrac{2(1-x^2)}{\sqrt{2-x^2}} \qquad f'(x)=0$ at $x=\pm 1$

$f'(x)$ is undef. at $x=\pm\sqrt{2}$

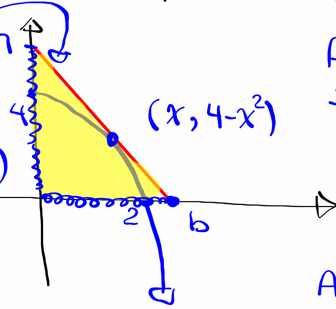
$f''(x) = \dfrac{2x(x^2-3)}{(2-x^2)^{3/2}} \qquad f''(x)=0 \rightarrow x=0,\ x=\pm\sqrt{3}$

using Wolfram Alpha.com $\qquad f''(x)$ is undef. @ $x=\pm\sqrt{2}$

x	$-\sqrt{2}$		-1		0		1		$\sqrt{2}$
$f'(x)$		$-$	•	$+$		$+$	•	$-$	
$f''(x)$		$+$		$+$	•	$-$		$-$	
$f(x)$		↘		↗		↗		↘	
			Min		I.P.		Max		



What is the smallest possible area of the triangle is cut off by QI whose hypotenuse is tangent to the parabola $y = 4 - x^2$?



$m = \frac{\text{Rise}}{\text{Run}} = \frac{-h}{b}$

$m = y'|_{(x, 4-x^2)}$

$m = -2x$

$-2x = \frac{-h}{b}$

$2bx = h$

Area of the triangle $= \frac{x\text{-int} \cdot y\text{-Int}}{2}$

$\text{Area} = \frac{bh}{2}$

$\text{Area} = \frac{b \cdot 2bx}{2}$

$\text{Area} = b^2 x$

$A(x) = b^2 x$

$A'(x) = b^2$

$0 \le x \le b$

$A(0) = b^2(0) = 0$ No area

$A(b) = b^3$

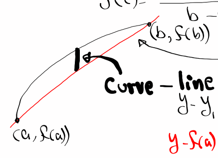
Finish this later

Mean-Value Theorem

$f(x)$ is cont. on $[a,b]$,

$f(x)$ is diff. on (a,b), There is at least one number c in (a,b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$



$m = \frac{f(b) - f(a)}{b - a}$

$y - y_1 = m(x - x_1)$

$y - f(a) = \frac{f(b) - f(a)}{b - a}(x - a)$

$F(x) = y = \frac{f(b) - f(a)}{b - a}(x - a) + f(a)$

$F(a) = f(a)$, $F(b) = f(b)$

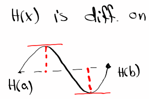
Vertical distance Curve − line

$f(x) - \left[\frac{f(b) - f(a)}{b - a}(x - a) + f(a)\right]$

$$\boxed{H(x) = f(x) - \frac{f(b) - f(a)}{b - a}(x - a) - f(a)}$$

$H(x)$ is cont. on $[a,b]$ $H(a) = 0$ $\Rightarrow H(a) = H(b)$

$H(x)$ is diff. on (a,b) $H(b) = 0$



By Rolle's Thrm $H'(c) = 0$

$H'(x) = f'(x) - \frac{f(b) - f(a)}{b - a} \cdot 1$

by Rolle's Thrm $H'(c) = 0$

$f'(c) - \frac{f(b) - f(a)}{b - a} = 0$

Conclusion of M.V.T. $\Rightarrow f'(c) = \frac{f(b) - f(a)}{b - a}$

$f(x) = 5 - 12x + 3x^2$, $[1, 3]$

$f(x)$ is polynomial, cont. & diff. everywhere

$f(1) = 5 - 12(1) + 3(1)^2 = -4$

$f(3) = 5 - 12(3) + 3(3)^2 = -4$

Cont. on $[1,3]$

diff. on $(1,3)$

$f(1) = f(3)$

by Rolle's Thrm

there is at least a number c in $(1,3)$

Such that $f'(c) = 0$

$f'(x) = -12 + 6x$

$f'(c) = -12 + 6c$

$-12 + 6c = 0 \rightarrow \boxed{c = 2}$

$f(x) = 2x^2 - 3x + 1$, $[0, 2]$

Cont. ✓ diff. ✓

$f(0) = 1$

$f(2) = 3$

$f'(x) = 4x - 3$

by MVT, there is at least one number c Such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$4c - 3 = \frac{3 - 1}{2 - 0} = \frac{2}{2} = 1$$

$4c - 3 = 1$

$\boxed{c = 1}$